The celebrated nodal domain theorem by Courant says that the number of nodal domains of an eigenfunction associated with a $k$-th eigenvalue of the Dirichlet Laplacian (eigenvalues listed in increasing order) should be less than or equal to $k$. Pleijel proved in 1956 that equality holds only for finitely many values of $k$. In this case we speak of the Courant sharp situation (this has a strong connection with the question of minimal spectral partitions).

If we look at the square, it is immediate that the first, second and fourth eigenvalues are Courant sharp. We would like to analyze the statement by Pleijel that there are no other cases. We also discuss some statements of Antonie Stern who was a PhD student of R. Courant and defended her PhD in 1924. Although we focus on her results concerning the square, let us mention that she also has similar results in the case of the sphere, fifty years before the paper of H. Lewy.